

$\overline{\text{MS}}$ quark distribution and dipole scattering matrix elements at high energy

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Abstract

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We discuss the operator relation that connects the renormalized quark distribution in the $\overline{\text{MS}}$ scheme with the Wilson-line correlator representing dipole scattering in the s-channel picture.

Parton picture and s-channel picture provide dual descriptions of high-energy hadron scattering. Based on the analysis [1], we here present the relation between the $\overline{\text{MS}}$ parton distribution for quarks and the dipole scattering matrix elements that enter in the s-channel approach to hard collisions. An introduction to the s-channel physical picture can be found in [2]. For motivation based on QCD at high parton density, see [3]-[5] and references therein.

Consider the quark distribution function, defined as hadronic matrix element of a certain product of operators [6],

$$f_q(x, \mu) = \frac{1}{4\pi} \int dy^- e^{ixP^+ y^-} \quad (1)$$

$$\times \langle P | \bar{\psi}(0) Q(0) \gamma^+ Q^\dagger(y^-) \psi(0, y^-, \mathbf{0}) | P \rangle_c$$

where ψ is the quark field, Q is the gauge link, and the subscript c is the instruction to take connected graphs. The matrix element (1) can be rewritten as the real part of a forward scattering amplitude [1], in which we think of the operator $Q^\dagger \psi$ as creating an antiquark plus an eikonal line in the minus direction, starting at distance y^- from the position of the target. The operator product

in Eq. (1) has ultraviolet divergences and requires renormalization.

Next suppose that the momentum fraction x is very small. This means that the typical distance y^- from the target to where the antiquark and the eikonal line are created is large, of order $1/(xP^+)$ (Fig. 1). In the proton rest frame this is far outside the proton [2, 3]. We describe the evolution of the parton system in the s-channel using the hamiltonian techniques [7]. This allows one to express the evolution operator in the high-energy approximation as an expansion in Wilson-line matrix elements, the leading term of which is the dipole term

$$\Xi(\Delta, \mathbf{b}) = \int [dP'] \langle P' | \frac{1}{N_c} \text{Tr} \{ 1 - F^\dagger(\mathbf{b} + \Delta/2) F(\mathbf{b} - \Delta/2) \} | P \rangle, \quad (2)$$

where, in the notation of [8], F is the non-

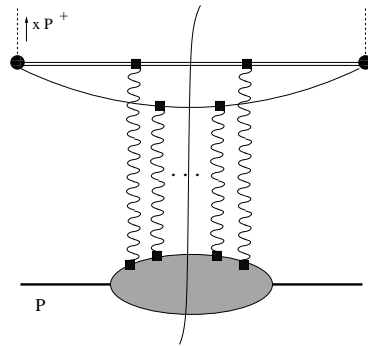


Figure 1: Parton distribution function for quarks in the s-channel picture.

abelian eikonal phase, Δ is the transverse

separation between the eikonals, and \mathbf{b} is the impact parameter.

In this s-channel representation, the quark distribution (1) is given by the coordinate-space convolution

$$xf_q(x, \mu) = \int d\mathbf{b} d\Delta u(\mu, \Delta) \Xi(\Delta, \mathbf{b}) - UV. \quad (3)$$

In [1] the explicit result is given for the function $u(\mu, \Delta)$ at one loop in dimensional regularization and for the counterterm $-UV$ of $\overline{\text{MS}}$ renormalization.

The one-loop $\overline{\text{MS}}$ result can be recast in a physically more transparent form in terms of a cut-off on the Δ integration region. This is defined by $\Delta\mu > a$, with a a parameter of order 1. To do this, the main point is to use the $-UV$ counterterm to cancel the ultraviolet divergence, and to determine a so as to cancel the finite remainder from the small- Δ integration region [1]. This yields

$$\begin{aligned} xf_{q/p}(x, \mu) &= \frac{N_c}{3\pi^4} \int d\mathbf{b} d\Delta \theta(\Delta^2 \mu^2 > a^2) \frac{\Xi(\Delta, \mathbf{b})}{\Delta^4} \\ &+ \frac{N_c}{3\pi^4} \int d\Delta \theta(\Delta^2 \mu^2 < a^2) \frac{R(\Delta)}{\Delta^4}, \end{aligned} \quad (4)$$

where R is the $\mathcal{O}(\Delta^4)$ remainder from the expansion of the \mathbf{b} -integral of Ξ near $\Delta = 0$, and a is a calculated number that accomplishes $\overline{\text{MS}}$ renormalization,

$$a = 2e^{1/6-\gamma} \approx 1.32657, \quad (5)$$

with γ the Euler constant. As long as μ is sufficiently large compared to the inverse hadron radius, the last term in the right hand side of Eq. (4) is suppressed by powers of μ , and one can write

$$xf_{q/p}(x, \mu) = \frac{N_c}{3\pi^4} \int d\mathbf{b} \frac{d\Delta}{\Delta^4} \theta(\Delta^2 \mu^2 > a^2) \Xi(\Delta, \mathbf{b}) \quad (6)$$

Eq. (6) is a remarkably simple formula. The power behavior $1/\Delta^4$ is perturbative,

and a is a renormalization scheme dependent coefficient, Eq. (5). The Wilson-line matrix element $\Xi(\Delta, \mathbf{b})$ receives contribution from both long distances and short distances. At large Δ (saturation region) [4], it has to be modeled or fit to data, while at small Δ it can be treated by a short distance expansion.

In particular, Ref. [1] uses one-loop renormalization-group evolution equations to relate $\Xi(\Delta, \mathbf{b})$ at small Δ to a well-defined integral of the gluon distribution function. This relation represents the expression (at one loop) of color transparency. It will be of interest to study this relation for the currently accessible x , jointly with Eq. (6), as a way to probe color transparency at the level of nucleon's parton distribution functions [1, 8].

The method described above can also be applied to quantities that have a more direct physical interpretation, e.g. structure functions. In particular, this is used in [9] to identify the power-like contributions to the Q^2 -derivative of F_2 that come from multiple gluon scatterings. These are enhanced as $x \rightarrow 0$, consistently with observations of approximate geometric scaling in low- x data [10]. Since the F_2 data used at present to extract the gluon density for $x < 10^{-2}$ do not have very high Q^2 , these power corrections can be relevant to the estimate of the theoretical accuracy on small- x pdf's for the LHC. If so, their effect is to be taken into account along with that [11] of higher-order $\ln x$ corrections.

We conclude by observing that the main applications of the results discussed in this article are to processes dominated by the sea quark distribution at small x . Besides collider applications, these include the estimate of cross sections for high-energy cosmic neutrinos. Recent work in [12] proposes incorporating unitarity corrections to DGLAP predictions [13, 14] for high-energy ν scattering, based on a parton-model fit to F_2 data that enforces the Froissart bound. Note that, if parton degrees of freedom are relevant at cos-

mic shower energies, we expect neutrino interactions to be dominated by the s-channel dynamics of the sea quark, as represented in Eq. (6) and Fig. 1.

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